Zeno on Infinite Division and 'Now' in Time

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In this paper, I take up three sorts of paradoxes by Zeno from the standpoint of infinite division and examine the solutions for each one. The final aim of this inquiry is to clarify the relation between time and now（khronos kai nun）through the examination of 'the Arrow'.

Regarding the first and second arguments against motion ('Dichotomy' and 'Achilles'), it is noteworthy that the runners are tacitly required to obey the procedures which Zeno suggests. And if we analyze these procedures from the viewpoint of algorithm, it becomes obvious that they constitute infinite loops. So, in these paradoxes the cause of runners' fail to reach the goal or catch up the tortoise is that they adopted incorrect algorithms. In the background of this problem, there is a difference between the motion and the operation. In operation we must obey some human rules or procedures, but in motion we have no such need. The running of Zeno's Paradox is the operation rather than the motion. A motion is able to complete the running of a *finite but endless* interval (i.e. a finite half-open interval) through reaching its upper bound, but any operation cannot be completed if the infinite steps of the operation are mapped into such an interval. Each step of the operation corresponds to one natural number, and because natural numbers are infinite and their set has on maximum and no upper bound, it is impossible for operation to complete the infinite steps through reaching such an element. Indeed, infinite operation has no definition of completion. And the each whole interval for the runners of these two paradoxes constitutes such half-open interval. To recognize this leads to the solution for the derivative paradox from Dichotomy (*Phys*.263a7-11). This derivative paradox is as follows. If a runner counts a number in order whenever he comes to the half of the remaining distance of racecourse, then he must have finished counting infinite numbers when he reach the goal, but it is contradiction. In truth, he doesn't finish such infinite operation when he reaches the goal, because there is no definition to be fulfilled of completing infinite operation. So, there is no contradiction.

We can resolve the paradoxes against plurality in this way. These paradoxes depend on completion of the dividing operation. If dividing process is not completed, the whole magnitude becomes neither infinite nor zero, but same as the beginning. And as I have mentioned above, the infinite operation cannot be completed. So, there is no contradiction.

The third paradox against motion (the Arrow) can be regarded as a paradox of the completed infinite division. For in the argument it is presupposed that time contains infinite numbers of now which, according to Aristotle, divide whole time into past and future. Aristotle thought this paradox depends on presupposition that time is composed of plenty of now. He understood the relation between time and now analogously to the relation between a line and a point. Enumerating the grounds for denying that the continuum is composed of points or nows, he emphasized that different geometrical points cannot contact (*Phys*.Z.1). If they contact, then they must be one and same point. Similar argument is valid for the relation between time and now. But the argument contrary to it seems to be valid as well, because any time must become now continuously in due course. I think this problem is caused by discrepancy between *now as present* and *now as moment*, and the concept of the latter was produced from the former by the operation of diminishing.